A. For $n=0, 2, 4, 6$, etc.

$\lambda=$	1	3	5	7	9	etc.
$x=$	1 2	4 5	7 8	10 11	13 14	

For $n=1, 3, 5, 7$, etc.

$\lambda=$	0	2	4	6	8	
$x=$	$-\frac{1}{2}$ $\frac{1}{2}$	$\frac{5}{2}$ $\frac{7}{2}$	$\frac{11}{2}$ $\frac{13}{2}$	$\frac{17}{2}$ $\frac{19}{2}$	$\frac{23}{2}$ $\frac{25}{2}$	

$y=n\sin\frac{1}{3}\pi$.

Similarly for negative values of n, λ.

MECHANICS.

96. Proposed by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

Two particles, subject to their mutual attraction and that of a fixed center, move in a plane containing the center. Find the motion under the law of the inverse square.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Take the center of force as origin. Let m, m_1, m_2 be the masses of the center of force and particles, respectively. r, r_1, ρ the distances of the particles from the center of force and from each other, respectively. (x, y), (x', y') the coördinates of the particles. The differential equations of motion of the two particles relative to the center of force are

$$\left.\begin{aligned}\frac{d^2x}{dt^2}&=-\frac{m+m_1}{r^3}x+\frac{x'-x}{\rho^3}m_2-\frac{m_2x'}{r_1^3}\\ \frac{d^2y}{dt^2}&=-\frac{m+m_1}{r^3}y+\frac{y'-y}{\rho^3}m_2-\frac{m_2y'}{r_1^3}\end{aligned}\right\}\ldots\ldots(1).$$

$$\left.\begin{aligned}\frac{d^2x'}{dt^2}&=-\frac{m+m_2}{r_1^3}x'+\frac{x-x'}{\rho^3}m_1-\frac{m_1x}{r^3}\\ \frac{d^2y}{dt^2}&=-\frac{m+m_2}{r_1^3}y'+\frac{y-y'}{\rho^3}m_1-\frac{m_1y}{r^3}\end{aligned}\right\}\ldots(2).$$

Where $\rho=\sqrt{[(x'-x)^2+(y'-y)^2]}$.

Multiply (1) by $2m_1\dfrac{dx}{dt}-2m_1\dfrac{m_1\dfrac{dx}{dt}+m_2\dfrac{dx'}{dt}}{m+m_1+m_2}$.

$$2m_1\frac{dy}{dt}-2m_1\frac{m_1\dfrac{dy}{dt}+m_2\dfrac{dy'}{dt}}{m+m_1+m_2}.$$

Multiply (2) by $2m_2\frac{dx'}{dt}-2m_2\dfrac{m_2\frac{dx'}{dt}+m_1\frac{dx}{dt}}{m+m_1+m_2}$,

$$\text{and } 2m_2\frac{dy'}{dt}-2m_2\frac{m_2\frac{dy'}{dt}+m_1\frac{dy}{dt}}{m+m_1+m_2}.$$

Adding the four products we get

$$2m_1\left(\frac{dx}{dt}\cdot\frac{d^2x}{dt^2}+\frac{dy}{dt}\cdot\frac{d^2y}{dt^2}\right)+2m_2\left(\frac{dx'}{dt}\cdot\frac{d^2x'}{dt^2}+\frac{dy'}{dt}\cdot\frac{d^2y'}{dt^2}\right)$$

$$-2\frac{m_1\frac{dx}{dt}+m_2\frac{dx'}{dt}}{m+m_1+m_2}\left(m_1\frac{d^2x}{dt^2}+m_2\frac{d^2x'}{dt^2}\right)$$

$$-2\frac{m_1\frac{dy}{dt}+m_2\frac{dy'}{dt}}{m+m_1+m_2}\left(m_1\frac{d^2y}{dt^2}+m_2\frac{d^2y'}{dt^2}\right)$$

$$-2m\left(\frac{m_1}{r^2}\frac{dr}{dt}+\frac{m_2}{r_1^2}\frac{dr_1}{dt}\right)-2\frac{d}{dt}\left(\frac{m_1m_2}{\sqrt{[(x'-x)^2+(y'-y)^2]}}\right)=0.$$

Integrating we get

$$m_1[(dx/dt)^2+(dy/dt)^2]+m_2[(dx'/dt)^2+(dy'/dt)^2]$$

$$-\frac{[(m_1dx/dt+m_2dx'/dt)^2+(m_1dy/dt+m_2dy'/dt)^2]}{m+m_1+m_2}$$

$$-2m\left(\frac{m_1}{r}+\frac{m_2}{r_1}\right)-\frac{2m_1m_2}{\sqrt{[(x'-x)^2+(y'-y)^2]}}=A,$$

or $mm_1[(dx/dt)^2+(dy/dt)^2]+mm_2[(dx'/dt)^2+(dy'/dt)^2]$

$$+m_1m_2[(dx'/dt-dxdt)^2+(dy'/dt-dy/dt)^2]$$

$$-2(m+m_1+m_2)\left[m\left(\frac{m_1}{r}+\frac{m_2}{r_1}\right)+\frac{m_1m_2}{\sqrt{[(x'-x)^2+(y'-y)^2]}}\right]=A.$$

The *vis viva* equation of motion.